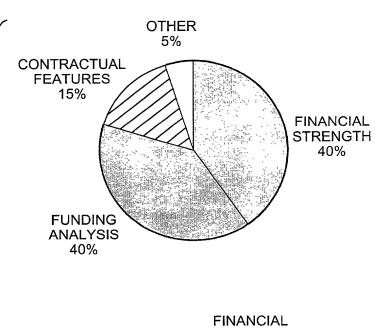
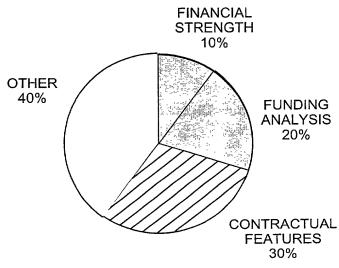


FIG. 1A





Docket No.: 3034.1000-001

Title: Method and System for Evaluation . . . Inventors: Daniel Johnson and Alok Mehta

# 2/25

FINANCIAL STRENGTH OF COMPANY	40
S&P RATING	6
WEISS RATING	6
BEST'S RATING	8
ASSET SIZE	10
STRENGTH OF BACKING FROM PARENT	10

FUNDING ANALYSIS	40
CASH FLOW REQUIRED FOR FUNDING	20
NET PRESENT VALUE OF A/T CASH FLOW AT X%	4
IRR ON COMPOSITE A/T CASH FLOW	4
A/T EFFECT ON EARNINGS @ YEAR 1	5
CUMULATIVE A/T EFFECT ON EARNINGS @ YEAR 5	5
EARNINGS CROSSOVER	2

CONTRACTUAL FEATURES	15
De MEC'ING PROVISIONS	3
MORTALITY CHARGE GUARANTEES	4
EXPENSE CHARGE GUARANTEES	4
BUYER RATING OF FUND CHOICES	2
BUYER RATING OF HISTORICAL FUND PERFORMANCE	2

OTHER	5
SUITABILITY OF UNDERWRITING OFFER	5

FIG. 1B

# NASA/TP-2007-214608



# Incorporation of Half-Cycle Theory Into Ko Aging Theory for Aerostructural Flight-Life Predictions

William L. Ko, Van T. Tran, and Tony Chen NASA Dryden Flight Research Center Edwards, California

National Aeronautics and Space Administration

Dryden Flight Research Center Edwards, California 93523-0273

Cover art: NASA Dryden Flight Research Center, photograph EC04-0029-17.
NOTICE  Use of trade names or names of manufacturers in this document does not constitute an official endorsement of such products or manufacturers, either expressed or implied, by the National Aeronautics and Space Administration.
Available from:  NASA Center for AeroSpace Information 7115 Standard Drive Hanover, MD 21076-1320 (301) 621-0390

# **CONTENTS**

ABSTRACT
NOMENCLATURE
INTRODUCTION4
THE B-52B AIRPLANE CARRYING THE HYPER-X LAUNCH VEHICLE 5
THE KO CLOSED-FORM AGING THEORY5Failure-Critical Structural Components5Stress/Load Equation6Operational Load Factor6Crack Size Determinations7The Ko Operational Life Equation8The Ko Operational Life Theory Flow Chart9
HALF-CYCLE CRACK GROWTH THEORY 10 The Walker Crack Growth Equation 10 The Half-Cycle Crack Growth Equation 11
CRACK GROWTH COMPUTER PROGRAM. 12
OPERATIONAL LIFE ANALYSIS14The B-52B and Pegasus Pylon Hooks14Flight Load Spectra15Crack Growth Calculations15Number of Operational Flights16
RESULTS
CONCLUSIONS
APPENDIX A - OPERATIONAL LIFE EQUATIONS
APPENDIX B - CRACK GROWTH COMPUTER PROGRAM
APPENDIX C - MATERIAL PROPERTIES

FIGURES	 	64											
REFERENCES													8

#### **ABSTRACT**

The half-cycle crack growth theory was incorporated into the Ko closed-form aging theory to improve accuracy in the predictions of operational flight life of failure-critical aerostructural components. A new crack growth computer program was written for reading the maximum and minimum loads of each half-cycle from the random loading spectra for crack growth calculations and generation of in-flight crack growth curves. The unified theories were then applied to calculate the number of flights (operational life) permitted for B-52B pylon hooks and Pegasus<sup>®</sup> adapter pylon hooks to carry the Hyper-X launching vehicle that air launches the X-43 Hyper-X research vehicle. A crack growth curve for each hook was generated for visual observation of the crack growth behavior during the entire air-launching or captive flight. It was found that taxiing and the takeoff run induced a major portion of the total crack growth per flight. The operational life theory presented can be applied to estimate the service life of any failure-critical structural components.

#### **NOMENCLATURE**

$\boldsymbol{A}$	crack location parameter ( $A = 1.12$ for a surface or edge crack)
a	depth (crack size) of semi-elliptic surface crack, in
$a_{i-1}$	crack size at the end of the $(i-1)$ -th half cycle, in
$a_c^o$	operational (final) crack size associated with operational
	load $V_{\text{max}}^o$ , in, $=\frac{Q}{\pi} \left( \frac{K_{IC}}{AM_k f \sigma^*} \right)^2 = \frac{a_c^p}{f^2}$
$a_c^p$	proof (initial) crack size associated with proof load $V^*$ , in, $=\frac{Q}{\pi} \left( \frac{K_{IC}}{\Delta M_{VC} \sigma^*} \right)^2$
$a_1$	crack size at the end of the first flight, in, $= a_c^p + \Delta a_1$
C	coefficient of Walker crack growth equation, $\frac{\text{in}}{\text{cycle}} \left( \text{ksi} \sqrt{\text{in}} \right)^{-m}$
c	half length of semi-elliptic surface crack, in
E(k)	complete elliptic function of the second kind, $=\int_0^{\pi/2} \sqrt{1-k^2 \sin^2 \phi} d\phi$
$F_1^*$	number of operational flights based on the first fight load data
f	operational load factor associated with the worst half cycle of random load
	spectrum, $=\frac{V_{max}^o}{V^*} = \frac{\sigma_{max}^o}{\sigma^*} = \sqrt{\frac{a_c^p}{a_c^o}}$
HXLV	Hyper-X launch vehicle

h thickness of hook, in  $K_{IC}$  mode I critical stress intensity factor, ksi  $\sqrt{\text{in}}$ 

 $K_{
m max}$  mode I stress intensity factor associated with  $\sigma_{
m max}$  , ksi  $\sqrt{
m in}$ 

 $\Delta K$  mode I stress intensity amplitude associated with stress amplitude, ( $\sigma_{\rm max} - \sigma_{\rm min}$ ), ksi  $\sqrt{\rm in}$ 

 $(K_{\text{max}})_i$ mode I stress intensity factor associated with  $(\sigma_{\text{max}})_i$  of *i*-th half cycle, ksi  $\sqrt{\text{in}}$  $\Delta K_i$ mode I stress intensity amplitude associated  $[(\sigma_{\max})_i - (\sigma_{\min})_i]$  of *i*-th half cycle, ksi √in i  $1, 2, 3, \dots$ , integer associated with the *i*-th half cycle  $1, 2, 3, \dots$ , integer associated with the j-th half cycle, or the j-th flight j modulus of elliptic function, =  $\sqrt{1 - \left(\frac{a}{c}\right)^2}$ k flaw magnification factor ( $M_k = 1$  for a shallow crack)  $M_k$ Walker stress intensity factor exponent associated with  $(K_{max})^m$ mnumber of stress cycles generated during the first flight  $N_1$ N partial stress cycles during flight (fraction of  $N_1$ ) Walker stress-ratio exponent associated with  $(1-R)^n$ nsurface flaw and plasticity factor, =  $[E(k)]^2 - 0.212 \left(\frac{\sigma^*}{\sigma_Y}\right)^2$ Qstress ratio associated with constant amplitude load cycle, =  $\frac{\sigma_{min}}{\sigma_{min}}$ R stress (or load) ratio associated with the worst half-cycle,  $=\frac{\sigma_{\min}^o}{\sigma_{\max}^o} = \frac{V_{\min}^o}{V_{\max}^o}$  $R^{o}$ stress ratio associated with the *i*-th half cycle,  $=\frac{(\sigma_{\min})_i}{(\sigma_{\max})_i}$  $R_i$ SRB/DTV solid rocket booster drop test vehicle  $V_A$ B-52B pylon front hook load, lb  $V_{BL}$ B-52B pylon left rear hook load. lb  $V_{BR}$ B-52B pylon right rear hook load, lb  $V_{PFL}$ Pegasus pylon front left hook load, lb  $V_{PFR}$ Pegasus pylon front right hook load, lb  $V_{PRL}$ Pegasus pylon rear left hook load, lb  $V_{PRR}$ Pegasus pylon rear right hook load, lb VA B-52B pylon front hook **VBL** B-52B pylon left rear hook **VBR** B-52B pylon right rear hook **VPFL** Pegasus pylon front left hook **VPFR** Pegasus pylon front right hook **VPRL** Pegasus pylon rear left hook **VPRR** Pegasus pylon rear right hook

V applied hook load, lb

 $V^*$  proof load for any hook, lb

 $V_{\max}^o$  maximum load of the worst cycle of random load spectrum, lb  $V_{\min}^o$  minimum load of the worst cycle of random load spectrum, lb  $\Delta a_1$  amount of crack growth induced at the end of the first flight, in

 $\Delta a$  amount of a partial crack growth at any time step during the flight, in

 $\delta a_i$  crack growth increment induced by the *i*-th half cycle, in

 $\eta$  stress/load coefficient, ksi/lb

 $\sigma^*$  tangential stress at critical stress point induced by the proof load  $V^*$ , ksi, =  $\eta V^*$ 

 $\sigma_A$  tangential stress at critical stress point of B-52B pylon front hook induced by  $V_A$ ,

ksi

 $\sigma_{BL}$  tangential stress at critical stress point of B-52B pylon rear left hook induced by

 $V_{BL}$ , ksi

 $\sigma_{BR}$  tangential stress at critical stress point of B-52B pylon rear right hook induced by

 $V_{BR}$ , ksi

 $\sigma_{PFL}$  tangential stress at critical stress point of Pegasus pylon front left hook induced by

 $V_{PFL}$ , ksi

 $\sigma_{PFR}$  tangential stress at critical stress point of Pegasus pylon front right hook induced

by  $V_{PFR}$ , ksi

 $\sigma_{PRL}$  tangential stress at critical stress point of Pegasus pylon rear left hook induced by

 $V_{PRL}$ , ksi

 $\sigma_{PRR}$  tangential stress at critical stress point of Pegasus pylon rear right hook induced by

 $V_{PRR}$ , ksi

 $\sigma_{\rm max}^o$  tangential stress at critical stress point associated with operational peak load,

 $V_{\rm max}^o$ , ksi

 $\sigma_U$  ultimate tensile stress, ksi

 $\sigma_Y$  yield stress, ksi

 $\sigma_{\max}$  maximum stress of constant amplitude loading cycles, ksi minimum stress of constant amplitude loading cycles, ksi

 $\sigma_t$  tangential stress along hook inner boundary, ksi

 $(\sigma_t)_{\text{max}}$  maximum value of  $\sigma_t$  at the stress critical point, ksi

 $au_U$  ultimate shear stress, ksi

 $\phi$  angular coordinate for semi-elliptic surface crack, rad

 $\theta_c$  angular location of critical stress point, deg

(); quantity associated with the *i*-th half cycle of random loading spectrum

()\* quantity associated with proof load

#### INTRODUCTION

The NASA Dryden B-52B (McDonnell Douglas, St. Louis, Missouri) launch airplane has been used to carry various types of flight research vehicles for high-altitude air-launching tests. The test vehicle is mated to the B-52B pylon through one L-shaped front hook and two identical L-shaped rear hooks. The L-shaped structural geometry will always induce tensile or compressive stress concentration depending on the loading direction (B-52 hooks can have only tensile stress concentrations). The inner curved boundary point of the hook where the tangential tensile stress reaches a maximum is called a critical stress point of the hook and is the potential fatigue crack initiation site.

During the early stages of the flight tests of the solid rocket booster drop test vehicle (SRB/DTV, 49,000 lb) (1983), the two old rear hooks (fabricated with 4340 steel) failed almost simultaneously during towing of the B-52B airplane carrying the SRB/DTV on a relatively smooth taxiway (low-amplitude dynamic loading). A microsurface crack at the critical stress point of each hook escaped detection because of surface masking by plating films. Those fatigue cracks could have been initiated from the past long period of flight test load cycling and the surface corrosion. If the hook failures had occurred during a takeoff run or during captive flight, a catastrophic accident might have occurred. This type of potential accident underscored the need for reliable and accurate calculations of the fatigue crack growths, which could thereby estimate the safe operational flight life of the hooks for each new flight test program.

Recently, the B-52B airplane has been used to carry the Hyper-X launching vehicle that airlaunches the X-43 hypersonic flight research vehicle for Mach 7–10 flight tests. The B-52B pylon hooks were intended to carry the total store weight of 40,000 lb (slightly lighter than the SRB/DTV weight 49,000 lb).

The safety of flight tests using B-52B pylon hooks to carry any drop-test vehicle [for example, the Hyper-X launching vehicle (HXLV)] hinges upon the structural integrity of the failure-critical structural components like B-52B pylon hooks and Pegasus® (Orbital Sciences Corporation, Dulles, Virginia) pylon hooks. It is, therefore, of vital importance to accurately determine the safe operational flight life for each of those failure-critical aerostructural components.

Earlier, Ko (refs. 1–6) developed several aging theories for predicting the operational flight life of airborne failure-critical structural components. The most accurate aging theory developed to date was the Ko closed-form aging theory (refs. 5, 6). In this report, the half-cycle crack growth theory will be incorporated into the Ko closed-form aging theory (refs. 5, 6) to improve the accuracy of operational life predictions of failure-critical airborne structural components. A special half-cycle crack growth computer program was written to calculate the crack growth needed for operational life predictions. The enhanced Ko closed-form aging theory was then applied to calculate the number of safe flights permitted for B-52B pylon hooks and Pegasus adapter pylon hooks to carry the HXLV for air-launching the X-43 hypersonic flight research vehicle.

The operational life theory presented in this report can also be applied to estimate the service life of any failure-critical structural components.

#### THE B-52B AIRPLANE CARRYING THE HYPER-X LAUNCH VEHICLE

Figure 1 shows the B-52B aircraft carrying the HXLV with the X-43 hypersonic flight research vehicle mated to its nose for air-launching flight tests at Mach 7–10. Because the Pegasus booster rocket has a delta wing which prevents the cylindrical booster body to nest closely under the B-52B pylon concave belly, a special adapter called the Pegasus adapter pylon (weighing 2,300 lb) is used to link the B-52B pylon hooks to the HXLV (weighing 37,700 lb). The Hyper-X launch vehicle is carried by the four identical Pegasus adapter pylon hooks, and the Pegasus adapter pylon is, in turn, carried by the B-52B pylon hooks using a double-shear pin to link to the front hook and through the Pegasus pylon adapter-shackles to connect to the two rear hooks of the B-52B pylon. The total weight then carried by the B-52B pylon hooks is 40,000 lb.

The double-shear pin is not fatigue-critical because there is no stress concentration problem. The two Pegasus pylon adapter shackles, however, are highly failure-critical because each shackle contains a rectangular hole with four, sharp, rounded corners in the upper part, and a circular hole in the lower part (ref. 8). Other failure-critical structural components identified are: the L-shaped B-52B front and two rear hooks and the four, identical L-shaped, Pegasus adapter pylon hooks (ref. 8).

The operational flight-life of all the pylon hooks will be analyzed because the actual loading spectra for those components are now available for the application of the half-cycle crack growth theory. The un-instrumented Pegasus adapter shackles were not analyzed because the actual loading spectra do not exist.

#### THE KO CLOSED-FORM AGING THEORY

The following section will describe the Ko closed-form aging theory. In the formulation of the Ko closed-form aging theory for aerostructural operational life predictions, the following steps are used.

# **Failure-Critical Structural Components**

Acomplex structure usually contains a certain number of failure-critical structural components, each of which contains a critical stress point. The critical stress point is a boundary point of the structural component where the tangential tensile stress concentration reaches a maximum, and is the potential fatigue crack initiation site. The operational life of the complex structure is then determined by the operational life of the worst failure-critical structural component having the shortest fatigue life (that is, the fastest crack growth rate at the critical stress point). Therefore, in the operational life analysis, the failure-critical structural components must be identified and their stress fields established.

# **Stress/Load Equation**

In the actual flight tests, the strain gages are usually installed in the vicinity of the critical stress point, and are calibrated to record the applied load (such as hook load). After the failure-critical structural components are identified, stress analysis must be performed for each critical structural component to establish the functional relationship between the applied load and the induced tangential stress at the critical stress point (refs. 7–9). For example, if  $V^*$  is the proof load, and if  $\sigma^*$  is the induced proof stress at the stress critical point, then the proof stress,  $\sigma^*$ , may be related to the proof load,  $V^*$ , through the following stress/load functional relationship in equation (1)

$$\sigma^* = \eta V^* \tag{1}$$

where  $\eta$  is defined as the stress/load coefficient, and is determined from the finite-element stress analysis of the critical structural component (refs. 7–9).

# **Operational Load Factor**

The next information needed in the operational life analysis is the operational load factor, f (<1), defined in equation (2) as

$$f = \frac{\sigma_{max}^o}{\sigma^*} = \frac{V_{max}^o}{V^*} < 1 \tag{2}$$

where  $\sigma_{\max}^o$  is the operational maximum stress at the critical stress point induced by the operational maximum load,  $V_{\max}^o$ , of the worst half-cycle of the random loading spectrum. The worst half-cycle is defined as the half-cycle with the maximum stress (load) amplitude, associated with the minimum stress ratio or load ratio as shown in equation (3)

$$(\sigma_{\max}^o - \sigma_{\min}^o) = \sigma_{\max}^o (1 - R^o) = \text{Maximum} \quad ; \quad R^o = \frac{\sigma_{\min}^o}{\sigma_{\max}^o} = \frac{V_{\min}^o}{V_{\max}^o} = \text{Minimum}$$
 (3)

where  $R^o$  is the stress (or load) ratio associated with the worst half-cycle. The worst half-cycle is to be searched out in light of condition (3) by means of a special load-factor-searching computer code embedded in the newly written crack growth computer program discussed in Appendix B. Keep in mind that the value of  $V_{\rm max}^o$  may not necessarily be the peak load of the entire flight-loading spectrum. Past flight load data showed that the operational maximum load,  $V_{\rm max}^o$ , usually occurred during the takeoff run because the ground effect induced the maximum crack growth rate.

#### **Crack Size Determinations**

In developing the Ko aging theory (refs. 5, 6), the proof (initial) and operational (final) crack sizes  $\{a_c^p, a_c^o\}$  at the critical stress point of the failure-critical structural component must be established first. The two crack sizes  $\{a_c^p, a_c^o\}$  are associated respectively with the proof and operational stresses  $\{\sigma^*, \sigma_{\text{max}}^o\}$  [or proof and operational peak loads  $\{V^*, V_{\text{max}}^o\}$ ], and are to be calculated from crack tip equations (4) and (5) based on the fracture mechanics (refs 1–4).

$$a_C^P = \frac{Q}{\pi} \left( \frac{K_{IC}}{AM_k \sigma^*} \right)^2 = \frac{Q}{\pi} \left( \frac{K_{IC}}{AM_k \eta V^*} \right)^2 \tag{4}$$

$$a_c^o = \frac{Q}{\pi} \left( \frac{K_{IC}}{AM_k \sigma_{max}^o} \right)^2 = \frac{Q}{\pi} \left( \frac{K_{IC}}{AM_k f \eta V^*} \right)^2 = \frac{a_c^p}{f^2}$$
 (5)

In equations (4) and (5),  $K_{IC}$  is the mode I critical stress intensity factor (material dependent), A is the crack location parameter (for a surface crack, A = 1.12, refs. 1–4),  $M_k$  is the flaw magnification factor (for a shallow surface crack,  $M_k = 1$ , refs. 1–4), and finally, Q is the surface flaw shape and plasticity factor. For an elliptic surface crack (surface length 2c, depth a), Q may be expressed as in equation (6) (refs. 1–4):

$$Q = [E(k)]^2 - 0.212 \left(\frac{\sigma^*}{\sigma_Y}\right)^2 \tag{6}$$

In equation (6),  $\sigma_Y$  is the yield stress, and E(k) is the complete elliptic function of the second kind defined in equation (7)

$$E(k) = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \phi} d\phi$$
 (7)

where  $\phi$  is the angular coordinate for a semi-elliptic surface crack, seen in fig. 2 (refs. 1–4), and k is the modulus of the elliptic function defined in equation (8)

$$k = \sqrt{1 - \left(\frac{a}{c}\right)^2} \tag{8}$$

Table 1 lists the input data for finding the values of E(k) from the complete elliptic integral table (ref. 10) for different crack aspect ratios a/2c. The values of Q were then calculated from equation (6). Table 1 lists only typical values of Q calculated for the worst stress ratio  $\sigma^*/\sigma_Y = 1$ .

a/2c	a/c	$k = \sqrt{1 - \left(a/c\right)^2}$	$\sin^{-1} k$ , deg.	<i>E</i> ( <i>k</i> )*	Q
0.1	0.2	0.979796	78.463041	1.0506	0.8918
0.2	0.4	0.916515	66.421822	1.1584	1.1299
0.25	0.5	0.866025	60.0	1.2111	1.2548
0.3	0.6	0.8	53.130102	1.2764	1.4172
0.4	0.8	0.6	36.869898	1.4181	1.7990
0.5	1.0	0.0	0.0	$\pi/2$	2.2554

Table 1. Key data for the calculations of Q, equation (6);  $\sigma^*/\sigma_Y = 1$ .

Figure 2 shows the value of Q plotted as a function of crack aspect ratio a/2c with stress ratio  $\sigma^*/\sigma_Y$  as a parameter. Remember that the values  $\{a/2c = 0.25, a/2c = 0.5\}$  listed in table 1 are respectively the aspect ratios of the actual initial surface cracks of the failed B-52B pylon old rear left and right hooks (ref. 7).

# The Ko Operational Life Equation

This section describes the basics of the Ko closed-form operational life equations (refs. 5, 6). In the formulation of Ko operational life equations, it was assumed that all the flights last for the same duration of time and induce identical random loading spectra. By representing the random loading spectra with the equivalent-constant-amplitude loading spectra so that the Walker crack growth equation (refs. 3, 4) may be applied, Ko (refs. 5, 6) formulated the closed-form operational life equation (as seen in equation (9) and derived in Appendix A) for the calculations of the number of flights,  $F_1^*$ , permitted for each failure-critical aerostructural component.

$$F_1^* = \frac{(a_c^p)^{1 - \frac{m}{2}} - (a_c^o)^{1 - \frac{m}{2}}}{(a_c^p)^{1 - \frac{m}{2}} - (a_1)^{1 - \frac{m}{2}}} = \frac{1 - f^{m-2}}{1 - \left(1 + \frac{\Delta a_1}{a_c^p}\right)^{1 - \frac{m}{2}}} \quad ; \quad a_1 = a_c^p + \Delta a_1$$
(9)

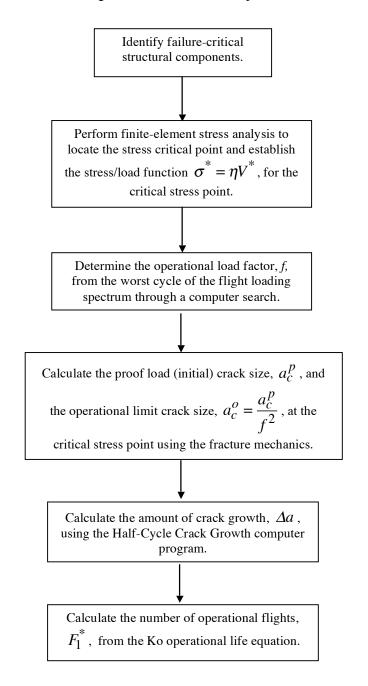
In equation (9),  $a_1 (= a_c^p + \Delta a_1)$  is the crack size at the end of the first flight, and  $\Delta a_1$  is the amount of crack growth induced by the first flight.

In equation (9), the known quantities are: the Walker stress-intensity-factor exponent m (refs. 3, 4), the load factor f [determined from equation (2)], and the proof (initial) and operational (final) crack sizes {  $a_c^p$ ,  $a_c^o$ } [calculated respectively from equations {(4), (5)}]. The only unknown is the crack growth,  $\Delta a_1$ , induced by the first flight. Therefore, the accuracy of the predicted operational flight life,  $F_1^*$ , from equation (9) is hinged upon the method of calculations used in determining the

<sup>\*</sup> Obtained from the complete elliptic integral table (ref. 10).

crack growth,  $\Delta a_1$ . The step-by-step processes required to use the Ko operational life equation (9) are shown in the following flow chart.

The Ko Operational Life Theory Flow Chart



#### HALF-CYCLE CRACK GROWTH THEORY

In the calculations of fatigue crack growth under random loading, there are several existing methods (ref. 11). For example,

- 1) Peak count method
- 2) Mean crossing peak count method
- 3) Range count method
- 4) Range-mean count method
- 5) Range pair count method
- 6) Level-crossing count method, and
- 7) Half-cycle method, etc. (ref. 11).

After reviewing the basics of those different theories, the half-cycle theory was chosen for the present crack growth calculations. The reason being that the half-cycle theory accounts every half-cycle of the random load spectrum without missing any secondary, small-amplitude half-cycles which do not even cross over the mean stress line (ref. 2). The second reason is that the predictions of fatigue life from the half-cycle theory compare fairly well with some existing experimental fatigue data (ref.11, pg. 211, ref. 12).

The half-cycle theory assumes that the amount of crack growth induced by each half-cycle of the random loading spectrum is considered as one-half of a complete cycle of a constant amplitude load spectrum with the same load amplitude. Figure 3 shows the resolutions of the random stress cycles into a series of half-cycles with different loading amplitudes (ref. 2). Under such assumption, the Walker crack growth equation may be used to calculate the incremental crack growth induced by each half-cycle with particular load amplitude.

# The Walker Crack Growth Equation

The Walker crack-growth equation for the constant amplitude load spectrum is given in equation (10) by

$$\frac{da}{dN} = C(K_{max})^m (1 - R)^n = C(\Delta K)^m (1 - R)^{n - m}$$
(10)

where C, m, n are material constants. The mode I stress intensity factor,  $K_{\text{max}}$ , mode I stress intensity amplitude,  $\Delta K$ , and the stress ratio, R are defined in equations (11), (12), and (13).

$$K_{\text{max}} = AM_k \sigma_{\text{max}} \sqrt{\frac{\pi a}{Q}}$$
 (11)

$$\Delta K = AM_k (\sigma_{\text{max}} - \sigma_{\text{min}}) \sqrt{\frac{\pi a}{Q}}$$
 (12)

$$R = \frac{\sigma_{\min}}{\sigma_{\max}} \tag{13}$$

where  $\{\sigma_{max}, \sigma_{min}\}$  are respectively the maximum and minimum stresses of the constant amplitude load spectrum. Equation (10) will now be modified to describe the half-cycle crack growth.

### The Half-Cycle Crack Growth Equation

In applying the half-cycle theory to calculate the crack growth induced by the random loading spectrum, it is assumed that the incremental amount of crack growth caused by each half-cycle with a particular load amplitude may be considered as a half-cycle of the constant amplitude loading spectrum with the same load amplitude. Therefore, the Walker crack growth equation, (10), may be used to calculate the incremental crack growth induced by each half-cycle of different load amplitude.

If the crack growth increment, da in equation (10), is set equal to the crack growth increment,  $\delta a_i$ , induced by the *i*-th (i = 1, 2, 3, ...) half cycle (i.e.,  $da = \delta a_i$ ), and the corresponding number of stress cycle increment, dN, is set equal to one half cycle (i.e., dN = 1/2), then the Walker crackgrowth equation (10) becomes the half-cycle crack growth equation for the calculations of half-cycle crack growth increment,  $\delta a_i$ . This half-cycle crack growth is expressed in equation (14).

$$\delta a_i = \frac{C}{2} \left[ (K_{max})_i \right]^m (1 - R_i)^n = \frac{C}{2} (\Delta K_i)^m (1 - R_i)^{n - m}$$
(14)

where  $\{(K_{\max})_i, \Delta K_i, R_i\}$  are respectively the values of  $\{K_{\max}, \Delta K, R\}$  [See equations (11)–(13)] associated with the *i*-th half-cycle given by equations (15), (16), and (17).

$$(K_{\text{max}})_i = AM_k(\sigma_{\text{max}})_i \sqrt{\frac{\pi a_{i-1}}{Q}}$$
 ;  $a_{1-1} = a_0 = a_c^p$  when  $i=1$  (15)

$$\Delta K_i = AM_k \left[ (\sigma_{\text{max}})_i - (\sigma_{\text{min}})_i \right] \sqrt{\frac{\pi a_{i-1}}{Q}}$$
(16)

$$R_i = \frac{(\sigma_{\min})_i}{(\sigma_{\max})_i} \tag{17}$$

where the subscript i (= 1, 2, 3, ....) is associated with the i-th half-cycle, and  $a_{i-1}$  is the cumulated crack size up to the (i-1)-th half-cycle. When i=1, the crack size  $a_{i-1}$  becomes  $a_{i-1}=a_{1-1}=a_0=a_c^p$ .

If N is any number of load cycles less than the total load cycles,  $N_1$ , induced by the first flight, then the amount of the partial crack growth,  $\Delta a$ , induced by the N load cycles may be obtained from the crack growth equation (18) by summing up all the previous half-cycle crack growth increments,  $\delta a_i$ , up to 2N (not N) cycles as

$$\Delta a = \sum_{i=1}^{2N} \delta a_i \quad ; \quad (N \le N_1)$$
 (18)

The summation process of the half-cycle crack growth according to equation (18) is graphically illustrated in fig. 4 (refs. 2–4). Equation (18) is used to calculate the increasing partial crack growths,  $\Delta a$ , with increasing numbers of cycles, N, (or flight time steps) for generating the data set for plotting the crack growth curve (crack growth as a function of flight time) for the critical structural component. When N reached the total number of cycles,  $N_1$ , ( $N = N_1$ ), equation (19) will give the total amount of crack growth,  $\Delta a_1$ , induced by the first flight. Namely,

$$(\Delta a)_{N=N_1} = \Delta a_1 = \sum_{i=1}^{2N_1} \delta a_i$$
 (19)

The value of  $\Delta a_1$ , calculated from equation (19) is to be used as input to equation (9) for the calculation of the number of operational flights  $F_1^*$  of the failure-critical structural component.

#### CRACK GROWTH COMPUTER PROGRAM

To carry out the summation of the half-cycle crack growth increment,  $\delta a_i$ , on the right-hand side of equation (18) or (19), a special crack growth computer program was written. (see Appendix B for details). To use this program and its results, it is necessary to perform the following steps.

1) Create a new data file containing only the required data from the time taxiing begins to the time of test vehicle drop (or the time of complete stop after captive touchdown)

- because a flight-test load data file is normally very big covering the ground-sitting portion. Keep in mind that the flight data is the load spectrum and not the stress cycles at the critical stress point.
- 2) Use a spike remove program to remove noises (spikes) from a flight load spectrum since spikes can add in erroneously big crack growth.
- 3) Run the crack growth program. This program prompts for an unc3 format input filename. Then, it prompts for some important InterRange Instrumentation Group B (IRIGB) times in milliseconds of start taxiing, takeoff run, cruise power, and drop (or captive stop). After getting all required input data, the crack growth computer program performs the following key functions.
  - a. Read the input flight load spectrum. For each channel (associated with each hook) in the input file, the program picks up the maximum and minimum loads for the *i*-th half-cycle,  $\{(V_{\text{max}})_i, (V_{\text{min}})_i\}$ . The half-cycle maximum load,  $(V_{\text{max}})_i$ , is determined when the load is bigger than the two adjacent loads; and conversely, the minimum load,  $(V_{\text{min}})_i$ , of the same half-cycle is determined when load is smaller than the two adjacent loads.
  - b. The loads  $\{(V_{\text{max}})_i, (V_{\text{min}})_i\}$  and their corresponding IRIGB times are saved in asc2 format output files. The names for the asc2 files are simple. For channel vap, the filename is sigma\_vap.asc2.
  - c. The loads  $\{(V_{\max})_i, (V_{\min})_i\}$  are then converted into the corresponding maximum and minimum stresses,  $\{(\sigma_{\max})_i, (\sigma_{\min})_i\}$ , of the *i*-th half cycle using equation (1).
  - d. Calculate the half-cycle crack growth increment,  $\delta a_i$ , using equation (14), and summing up  $\delta a_i$  over different numbers of load cycles, N (or a time step), to generate a data set of different partial crack growths,  $\Delta a$ , from equation (18).
  - e. Compute the total crack growth,  $\Delta a_1$ , from equation (19) for the entire flight (from the time of start taxiing to the time of drop or captive stop) for approximately every minute. The times in minutes (zero at start taxiing time) and its corresponding  $\Delta a_1$  are saved in an unc3 output file.
  - f. Determine the worst half-cycle from the loading spectrum during takeoff run and cruise power using the criterion of equation (3) and obtain the operational maximum load,  $V_{\max}^o$ , of the worst half-cycle.
  - g. Compute the load factor, *f*, from equation (2).
  - h. Calculate the number of operational flights,  $F_1^*$  from equation (9) based on the first flight load data.

- i. Generate a summary report in text format. This file contains the name of each B-52B hook in the input file, its total crack growth for the first flight,  $\Delta a_1$ , its number of operational flights,  $F_1^*$ , its operational load factor f, its worst half cycle maximum load  $V_{\max}^o$ , its worst half-cycle minimum load,  $V_{\min}^o$ , and the corresponding IRIGB time. It also has the values of the numerator and the denominator that are used to calculate  $F_1^*$ .
- j. Print on screen the names of the crack growth output file and the summary file.
- 4) Convert the crack growth file to asc2 format and then to Mircosoft (Redmond, Washington) Excel format.
- 5) Graphically plot  $\Delta a$  as a function of flight time in minutes using Excel.

#### **OPERATIONAL LIFE ANALYSIS**

The Ko aging theory with the half-cycle crack growth theory incorporated, will now be applied to calculate the operational life spans of the three B-52B pylon hooks, and the four Pegasus adapter pylon hooks carrying the HXLV.

Two types of flights were analyzed: 1) air-launching flight, 2) captive flight. The air-launching flight lasted for 106 minutes, counted from the time of B-52B break release for taxiing until the time of air launching (dropping of the HXLV). The captive flight (no air-launching of the HXLV) lasted for 191 minutes, counted from the time of B-52B break release for taxiing and takeoff until the time of complete stop after captive landing.

The purpose of the analysis is to compare the crack growths,  $\Delta a_1$ , induced by the first air-launching and first captive flight, and find out how many air-launching flights will be consumed by each captive flight. The actual flight loading data were used for the operational life calculations.

# The B-52B and Pegasus Pylon Hooks

Figures 5–10, taken from reference 8, respectively show the geometry of B-52B pylon hooks (figs. 5, 7), and a typical Pegasus adapter pylon hook (fig. 9). The tangential tensile stress distribution over the inner boundary of each hook, obtained from finite-element stress analysis (figs. 6, 8, 10), is also shown, together with the locations of the critical stress points and the stress/load relationships indicated. The stress/load coefficients,  $\eta$ , for B-52B pylon hooks and Pegasus adapter pylon hooks established from the finite-element stress analysis are summarized in table 2 (taken from ref. 8).

Table 2. Proof loads,  $V^*$ , and stress/load coefficients,  $\eta$ , for B-52B pylon hooks and Pegasus adapter pylon hooks.

Hooks	$V^*$ , lb	$\eta$ , ksi/lb
VA	36,500	7.3522×10 <sup>-3</sup>
VBL	57,819	5.8442×10 <sup>-3</sup>
VBR	57,819	5.8442×10 <sup>-3</sup>
VPFL	75,000	2.4459×10 <sup>-3</sup>
VPFR	75,000	2.4459×10 <sup>-3</sup>
VPRL	75,000	2.4459×10 <sup>-3</sup>
VPRR	75,000	$2.4459 \times 10^{-3}$

The stress/load coefficients,  $\eta$ , listed in table 2 are to be input to the crack-growth computer program to convert the loading spectrum of each hook into the stress cycles associated with the critical stress point using equation (1).

# Flight Load Spectra

Figures 11–17 respectively show the flight load spectra of the B-52B pylon hooks and the Pegasus adapter pylon hooks carrying the HXLV during the takeoff run of the first air-launching flight. The location of the worst half-cycle and the value of the load factor, f, are indicated in each figure. The worst half-cycle was located by means of the crack growth computer program searching over the takeoff run portion of each flight load spectrum, and then finding the value of the operational maximum load,  $V_{\rm max}^o$  (= $\sigma_{\rm max}^o/\eta$ ), of the worst half-cycle with minimum load ratio or stress ratio,  $R^o$  expressed in equation (20),

$$R^{o} = \frac{\sigma_{\min}^{o}}{\sigma_{\max}^{o}} = \frac{\eta V_{\min}^{o}}{\eta V_{\max}^{o}} = \frac{V_{\min}^{o}}{V_{\max}^{o}} = \text{ minimum}$$
 (20)

The value of  $V_{\text{max}}^o$  (or  $\sigma_{\text{max}}^o$ ) was then used to calculate the load factor, f, for each hook using equation (2).

#### **Crack Growth Calculations**

The material properties of B-52B pylon hooks and Pegasus adapter pylon hooks listed in Appendix C were used for the crack growth calculations. In the present crack growth calculations, the surface crack (A = 1.12) at the critical stress point of each hook was assumed to be a very shallow ( $M_k = 1$ ) semi-elliptic surface crack. Only one aspect ratio, a/2c = 1/4 (Q = 1.2548, table

1) was considered. As mentioned earlier, the value a/2c = 1/4 is the aspect ratio of the microsurface crack which caused the failure of a B-52B pylon old rear left hook (ref. 7). The crack-growth computer program was then used to read the values of  $\{(V_{\text{max}})_i, (V_{\text{min}})_i\}$  for each half-cycle over the loading spectrum, and converted them into the corresponding stresses  $\{(\sigma_{\text{max}})_i, (\sigma_{\text{min}})_i\}$  through equation (1) using the  $\eta$  values given in table 2 to calculate the half-cycle crack growth increment,  $\delta a_i$ , using equation (14). Finally,  $\delta a_i$  are summed up to different desired cycles (or time steps) to obtain partial crack growth,  $\Delta a$ , using equation (18) for generating a data set for plotting the crack growth curve for each hook. This process is graphically illustrated in fig. 3 and 4.

# **Number of Operational Flights**

After the total crack growth,  $\Delta a_1$ , induced by the first flight is calculated from equation (19) with the aid of the crack-growth computer program, the operational life equation (9) was then used to calculate the safe number of operational flights,  $F_1^*$ , allowed for the B-52B pylon hooks and Pegasus adapter pylon hooks to carry the HXLV for air-launching and captive flights.

#### RESULTS

The following sections discuss the results of the operational life analysis of the B-52B pylon hooks and the Pegasus adapter pylon hooks carrying the HXLV. This analysis uses the Ko aging theory and is aided by the half-cycle crack growth calculation method.

#### **Crack Growth Curves**

The crack growth curve is a very powerful tool for visually observing the crack growth behavior at the critical stress point of each failure-critical component. The crack growth curve for each hook was generated for the following two types of flights: air-launching flight and captive flight.

### **Air-Launching Flight**

Figures 18–20 respectively show the crack growth curves generated for the three B-52B pylon hooks. Those crack growth curves were calculated from equation (18) with the crack growth summation carried out by the crack-growth computer program using the first air-launching flight data. Notice that the crack growth rate for each hook is quite rapid during taxiing because of ground effect, and became more accelerated (illustrated by a steeper slope on the graph) during the takeoff run as the ground-induced vibrations intensified. Once airborne, the ground effect diminished and, therefore, the crack growth rate slowed down considerably and stayed relatively constant (except for encountering wind gusts) until air-launching. The crack growth curve for the B-52B front hook (VA, fig. 18) exhibits the steepest takeoff-run slope as compared with the B-52B two rear hooks (VBL and VBR, figs. 19, 20). The rapid crack growth of the B-52B front hook during the takeoff run could be attributed in part to the overhanging effect of the X-43, which is at a forward distance from the front hook. For the three B-52B pylon hooks, (VA, VBL, VBR), taxiing and takeoff runs

combined induced approximately 65, 51, and 41 percent of the respective total crack growth,  $\Delta a_1$ , per flight.

Figures 21–24 respectively show the crack growth curves for the four Pegasus adapter pylon hooks (VPFL, VPFR, VPRL, and VPRR). Those crack growth curves were generated from the crack growth computer program in carrying out the summation in equation (18) using the first air-launching-flight load data. The crack growth behavior of the Pegasus adapter pylon hooks is similar to that of the B-52B hooks, but with lower crack growth rates, especially during cruise flight. For the four Pegasus adapter pylon hooks (VPFL, VPFR, VPRL, and VPRR), the taxiing and takeoff run combined induced approximately 45, 60, 64, and 41 precent of the respective total crack growth,  $\Delta a_1$ , per flight.

# **Captive Flight**

Figures 25–27 respectively show the crack growth curves generated for the three B-52B pylon hooks (VA, VBL, and VBR) using the first captive-flight data. These crack growth curves were calculated from equation (18) with the crack growth summation carried out by the crack-growth computer program. Notice that, for each B-52B pylon hook, the amounts of crack growth and the crack growth rates (shown by slopes on the graphs) during the takeoff phase and the landing phase are quite similar. During the smooth cruise phase, the B-52B airplane encountered only two minor wind gusts (gust 1 and gust 2). The cruising crack growth rate of the front hook (VA, fig. 25) is much slower than those of the two rear hooks (VBL and VBR, figs. 26, 27). At the end of the cruise, three gusts were encountered by the B-52B airplane. The most severe, gust 5 coinciding with the B-52B maneuver, caused the crack growth rate for each hook to increase rapidly (portrayed by steeper slopes). For these three hooks, the fastest crack growth rates occurred during both the takeoff phase and landing phase because of severe ground effects. For the three B-52 hooks (VA, VBL, VBR), the takeoff phase and the landing phase combined contributed approximately 67, 54, and 51 percent of the respective total crack growth,  $\Delta a_1$ , per flight. The crack growth rate of the outboard right rear hook (VBR) during cruising flight is slightly faster than that of the inboard left rear hook (VBL). This phenomenon was also observed in the air-launching flight-test case (figs. 19, 20).

Figures 28–31 respectively show the crack growth curves generated for the Pegasus adapter pylon hooks (VPFL, VPFR, VPRL, and VPRR) by the crack growth computer program. The program carried out the summation of half-cycle crack growths, calculated by equation (18), associated with the first captive-flight load spectra. The crack growth curves of the Pegasus adapter pylon hooks are similar to those of the B-52B hooks, but with lower crack growth rates, especially during cruise flight. For the four Pegasus adapter pylon hooks (VPFL, VPFR, VPRL, and VPRR), the takeoff phase and landing phase combined induced nearly 51, 59, 71, and 51 percent of the respective total crack growth,  $\Delta a_1$ , per flight.

# **Number of Operational Flights**

The number of possible operational flights for each of the B-52B pylon hooks and of Pegasus adapter pylon hooks (carrying the HXLV) were calculated from the operational life equation, (9). Flight test data was obtained from two types of test flights, air-launching and captive.

# **Air-Launching Flight**

For the air-launching flight, which lasted for 106 minutes, the key input and output data generated for different hooks are listed in table 3 for crack geometry a/2c = 0.25 (Q = 1.2548).

Table 3. Key data for the B-52B airplane carrying the Hyper-X launch vehicle (total weight: 40,000 lb); 106-min air-launching flight; a / 2c = 0.25 (Q = 1.2548).

Hooks	$V^*$ , lb	$V_{\max}^o$ , lb	f	$a_c^p$ , in	$\Delta a_1$ , in	$F_1^*$ , flights
VA	36,500	18,065	0.4949	0.0691	1.9258×10 <sup>-4</sup>	304
VBL	57,819	23,227	0.4017	0.0429	2.5367×10 <sup>-4</sup>	$186^{\dagger}$
VBR	57,819	18,906	0.3270	0.0429	2.5734×10 <sup>-4</sup>	203
VPFL	75,000	34,367	0.4582	0.1455	1.6680×10 <sup>-4</sup>	873
VPFR	75,000	34,623	0.4616	0.1455	$1.8326 \times 10^{-4}$	790
VPRL	75,000	21,179	0.2824	0.1455	$1.4053 \times 10^{-4}$	$1,323^{\dagger\dagger}$
VPRR	75,000	21,413	0.2855	0.1455	$1.5441 \times 10^{-4}$	1,200

<sup>†</sup> Shortest operational life, †† Longest operational life

Table 3 shows that, among the three B-52B pylon hooks, the rear left hook (VBL) has the shortest life (186 flights), and the front hook (VA) has the longest life (304 flights). Although the crack growths for VBL and VBR are quite close, the higher value of f for VBL (f = 0.4017) caused the operational life of VBL to be shorter than VBR (f = 0.3270).

Among the four Pegasus pylon adapter hooks, the front right hook (VPFR) has the shortest life (790 flights), and the rear left hook (VPRL) has the longest life (1323 flights).

# **Captive Flight**

For the captive flight which had a duration of 191 minutes, the resulting key input and output data for different hooks are listed in table 4 for a/2c = 0.25 (Q = 1.2548).

Table 4. Key data for the B-52B airplane carrying the Hyper-X launce	ch vehicle
(total weight: 40,000 lb); 191-min captive flight; $a / 2c = 0.25$ ( $Q = 0.25$ )	1.2548).

Hooks	$V^*$ , lb	$V_{\max}^o$ , lb	f	$a_c^p$ , in	$\Delta a_1$ , in	$F_1^*$ , flights
VA	36,500	17,171	0.4704	0.0691	6.7226×10 <sup>-4</sup>	91
VBL	57,819	21,616	0.3739	0.0429	$7.4446 \times 10^{-4}$	$83^{\dagger}$
VBR	57,819	17,875	0.3092	0.0429	5.8556×10 <sup>-4</sup>	92
VPFL	75,000	33,482	0.4464	0.1455	2.1151×10 <sup>-4</sup>	477
VPFR	75,000	34,137	0.4552	0.1455	$3.3859 \times 10^{-4}$	433
VPRL	75,000	22,565	0.3009	0.1455	$3.1070 \times 10^{-4}$	$586^{\dagger\dagger}$
VPRR	75,000	21,087	0.2812	0.1455	3.3090×10 <sup>-4</sup>	563

<sup>†</sup> Shortest operational life, †† Longest operational life

Table 4 shows that, like the air-launching flight, the life of the B-52B pylon rear left hook (VBL) at 83 flights is shorter than the identical rear right hook (VBR) at 92 flights because of higher values of  $\{\Delta a_1, f\}$ . Among the four identical Pegasus pylon hooks, the front right hook has the shortest life (433 flights), and the rear left hook (VPRL) has the longest life (586 flights). Also, note from table 4 that crack growths,  $\Delta a_1$ , induced by the captive flight are approximately 2–3 times larger than  $\Delta a_1$  induced by the air-launching flight, therefore, the flight life of each hook is reduced.

Table 5 compares the operational flight life of each hook undergoing air-launching flight and captive flight. The ratio  $\frac{(F_1^*)_{\text{Air-launching}}}{(F_1^*)_{\text{Captive}}}$  will then give the number of air-launching flights consumed by each captive flight.